NONSTATIONARY MOTION OF DROPS UNDER THE ACTION OF CAPILLARY AND MASS FORCES

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The problem of nonstationary drift of drops under the action of time-varying thermocapillary and mass forces, with a constant temperature gradient given at infinity, was treated in [1] within the Stokes approximation. It was assumed that the motion starts with the fluid at rest, and occurs with axial symmetry.

Small Peclet numbers were assumed in [1] in solving the thermal problem. Therefore, the temperature field and the distribution of the surface tension coefficient generated by it along the surface of the drop have been independent of the fluid motion. A similar independence also occurs in the general case of temperature and concentration fields (or any other factor on which, generally speaking, the surface tension depends), when the Peclet number is small and the convective heat or material transport can be neglected. Formulating this situation, as was done in [2] for the stationary case, the nonstationary drift of drops is treated in the present paper, when the surface tension coefficient is given as a known function of time and of the points on the surface of the drop. Axial symmetry was not assumed in that case, and the integrodifferential equation derived here within the Stokes approximation for the velocity of motion of the drop center of mass has a vector form. To apply the results obtained, this function of surface tension must be calculated in each specific case.

Let a drop of a viscous incompressible fluid be located in another viscous incompressible fluid, filling all space. Consider the problem of nonstationary drop motion under the action of mass and capillary forces. It is assumed that the mass force is caused by a gravitational field with acceleration $\mathbf{g}(t)$, arbitrarily varying with time t, both in magnitude and in direction. The surface tension coefficient $\sigma(t, \theta, \varphi)$ is given as function of coordinates θ, φ on the drop surface and of time, and its nonconstancy leads to the occurrence of capillary forces.

In the solution we use the Stokes approximation, and assume that the motion starts from a fluid state of rest, with constant densities ρ_i and dynamic viscosities μ_i of the fluids (here and in the following the subscripts i = 1, 2 refer to the exterior fluid and to the drop, respectively). It is assumed that the surface tension is quite large, so that the drop shape is only insignificantly nonspherical. In that case, in particular, $\sigma^0 \gg \sigma'$, where σ^0 is of the order of the quantity, and σ' is of the order of variation of the surface tension coefficient on the surface of the drop [both these quantities are determined by the function $\sigma(t, \theta, \varphi)$].

The treatment is carried out in a noninertial reference system, moving translationally with the drop center of mass. The radius r is measured from it in the spherical coordinate system used here (r, θ, φ) (θ, φ are the meridional and azimuthal angles).

In the mathematical formulation of the problem we use the following dimensionless quantities. We choose the drop radius a to be the scale of distance, the quantity σ'/μ_1 for velocity, $\rho_1 a^2/\mu_1$ for time, and σ'/a for pressure. Instead of the gravity acceleration force $\mathbf{g}(t)$ we introduce the dimensionless quantity $\eta(t) = \rho_1 a^2 g/\sigma'$. For the dimensionless time and radius we use the preceding notation. In the dimensionless variables the equations, initial and boundary conditions for the radial component of the field velocity u_{ir} and pressure p_i (i = 1, 2) are then represented in the form

$$\frac{\partial u_{1r}}{\partial t} = -\frac{\partial p_1}{\partial r} + \frac{1}{r}\Delta(ru_{1r}); \tag{1}$$

$$\nu^{-1}\frac{\partial u_{2r}}{\partial t} = -\beta^{-1}\frac{\partial p_2}{\partial r} + \frac{1}{r}\Delta(ru_{2r}); \tag{2}$$

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$$\Delta p_i = 0 \ (i = 1,2); \tag{3}$$

$$r \to \infty, \ u_{1r} \to -(\mathbf{u} \cdot \mathbf{e}_r), \ p_1 \to 0; \tag{4}$$

$$r \to 0, \ u_{2r} < \infty, \ p_2 < \infty; \tag{5}$$

$$r = 1, \ u_{1r} = u_{2r} = 0; \tag{6}$$

$$\partial(r^2 u_{1r})/\partial r = \partial(r^2 u_{2r})/\partial r; \tag{7}$$

$$(2\partial/\partial r - \partial^2/\partial r^2)\{r^2(u_{1r} - \beta u_{2r})\} + \Delta_\Gamma \gamma = 0; \tag{8}$$

$$\int\limits_{\Gamma_1} \left\{ -p_1 + p_2 + (\rho - 1)\left(\eta - \frac{d\mathbf{u}}{dt}\right) \cdot \mathbf{e}_r + 2\frac{\partial}{\partial r}(u_{1r} - \beta u_{2r}) - 2\gamma \right\} \Phi_1 \, d\Gamma = 0; \tag{9}$$

$$\begin{gathered} t = 0, \ u_{ir} = 0 \ (i = 1,2), \ \mathbf{u} = 0, \\ \rho = \rho_2/\rho_1, \ \beta = \mu_2/\mu_1, \ \nu = \nu_2/\nu_1, \ \nu_i = \mu_i/\rho_i \quad (i = 1,2). \end{gathered} \tag{10}$$

Equations (1), (2) are the nonstationary Stokes equations for the radial velocity components; in that case the angular components $u_{i\theta}$, $u_{i\varphi}$ have already been eliminated from them by means of the equation of continuity for the incompressible fluid

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u_{ir}) + \frac{1}{r \sin\theta}\frac{\partial}{\partial\theta}(\sin\theta u_{i\theta}) + \frac{1}{r\sin\theta}\frac{\partial u_{i\varphi}}{\partial\varphi} = 0. \tag{11}$$

Equations (3) are obtained by taking the divergence of both sides of the nonstationary Stokes equations and taking into account (11). The behavior at infinity is reflected in condition (4), where $\mathbf{u}(t)$ is the dimensionless velocity of drop motion, and e_r is the unit radial vector. The absence of singularities at the origin of coordinates is noted in Eq. (5). The boundary conditions at the surface of the drop are represented in (6)-(9).

Condition (7), as well as (8), in which $\gamma = (\sigma(t, \theta, \varphi) - \sigma^0)/\sigma'$, and the operator Δ_Γ, the angular part of the Laplacian, are obtained as follows. We write the continuity conditions of the tangential velocity components and the tangential stresses at the drop surface:

$$u_{1\theta} = u_{2\theta}, \ \left(\frac{\partial u_{1r}}{\partial\theta} + \frac{\partial u_{1\theta}}{\partial r} - u_{1\theta}\right) - \beta\left(\frac{\partial u_{2r}}{\partial\theta} + \frac{\partial u_{2\theta}}{\partial r} - u_{2\theta}\right) + \frac{\partial\gamma}{\partial\theta} = 0; \tag{12}$$

$$u_{1\varphi} = u_{2\varphi}, \ \left(\frac{1}{\sin\theta}\frac{\partial u_{1r}}{\partial\varphi} + \frac{\partial u_{1\varphi}}{\partial r} - u_{1\varphi}\right) - \beta\left(\frac{1}{\sin\theta}\frac{\partial u_{2r}}{\partial\varphi} + \frac{\partial u_{2\varphi}}{\partial r} - u_{2\varphi}\right) + \frac{1}{\sin\theta}\frac{\partial\gamma}{\partial\varphi} = 0. \tag{13}$$

We apply to equality (12) the operator $(\sin\theta)^{-1}(\partial/\partial\theta)\sin\theta$, and to (13) the operator $(\sin\theta)^{-1}\partial/\partial\varphi$, and add the results in pairs. Eliminating then the angular velocity components by means of (11), with account of (6) we obtain (7) and (8).

In the boundary condition of the balance of normal stresses (9) one multiplies by an arbitrary spherical function of first order $\Phi_1(\theta, \varphi)$ and integrates over the whole surface of the drop $\Gamma_1(d\Gamma = \sin\theta d\theta d\varphi)$. The meaning of this integration consists of the following. Since in the present study we neglect the deviation of the drop shape from spherical, the boundary condition for the normal stresses must be omitted, replacing it by the cruder condition of balance of forces acting on the drop as a whole (as was done, in particular, in [3]), which is equivalent to a large extent to the integration performed here (in the calculation of forces acting on the drop one must also integrate the stress over the whole surface of the drop).

If, however, one writes down the direct condition for the normal stresses, one must inevitably contain in it a term proportional to the deviation $\varepsilon(\theta, \varphi)$ of drop shape from the spherical r = 1 (see, for example, [3]). In expanding this term in spherical functions

$$\Phi_n = \sum_{k=0}^{n} (a_{nk}\cos k\varphi + b_{nk}\sin k\varphi)\, P_n^k(\cos\theta) \quad (n = 0, 1, 2, \ldots), \tag{14}$$

where $P_n^k(z)$ is the associated Legendre function, functions of zeroth (n = 0) and first (n = 1) orders must be absent, since the volume of the drop does not change with deformation, and the origin of coordinates remains primarily at the center of mass of the drop. Therefore, in performing an integration such as (9) the term corresponding to shape deviation from spherical vanishes, and the description (9) is quite correct. We note that for our treatment condition (9) is more convenient than the condition of balance of forces acting on the drop as a whole, ordinarily used under these circumstances. The fact that the motion starts from the state of rest is expressed in the initial conditions (10).

The main purpose of the present study is to express the velocity of drop motion $\mathbf{u}(t)$ in terms of known characteristics of the mass and capillary forces $\mathbf{g}(t)$ and $\sigma(t, \theta, \varphi)$, which is implemented by solving the problem (1)-(10). As will be seen from the following, to achieve this purpose it is not quite necessary to find the velocity field in complete form. For example, it is not necessary to treat the angular velocity components, but restrict the discussion to the radial component only and to the pressure, for which the problem (1)-(10) is formulated.

We denote the Laplace transforms of all time-dependent functions appearing in problem (1)-(10) by the preceding symbols with asterisks. In that case, instead of the time t, the given functions depend on the Laplace transform parameter s.

Taking into account the initial conditions (10), from (1)-(9) we obtain the following problem for the transformed quantities:

$$su_{1r}^* = -\frac{\partial p_1^*}{\partial r} + \frac{1}{r}\Delta\left(ru_{1r}^*\right), \tag{15}$$

$$\nu^{-1}su_{2r}^* = -\beta^{-1}\frac{\partial p_2^*}{\partial r} + \frac{1}{r}\Delta\left(ru_{2r}^*\right),\ \Delta p_i^* = 0 \quad (i = 1, 2);$$

$$r\to\infty,\ u_{1r}^*\to\Phi_1^{(u)},\ p_1^*\to 0, \tag{16}$$

$$r\to 0,\ u_{2r}^*<\infty,\ p_2^*<\infty,\ r=1,\ u_{1r}^*=u_{2r}^*=0,$$

$$\partial\left(r^2u_{1r}^*\right)/\partial r = \partial\left(r^2u_{2r}^*\right)/\partial r,$$

$$(2\partial/\partial r - \partial^2/\partial r^2)\left\{r^2\left(u_{1r}^* - \beta u_{2r}^*\right)\right\} + \Delta_\Gamma\gamma^* = 0;$$

$$\int\limits_{\Gamma_1}\left\{-p_1^* + p_2^* + (\rho-1)\left(s\Phi_1^{(u)} + \Phi_1^{(\eta)}\right) + 2\frac{\partial}{\partial r}\left(u_{1r}^* - \beta u_{2r}^*\right) - 2\gamma^*\right\}\Phi_1\, d\Gamma = 0. \tag{17}$$

The first order spherical functions appearing in (16), (17) are determined by the relations

$$\Phi_1^{(u)} = -\mathbf{u}^*\cdot\mathbf{e}_r,\ \Phi_1^{(\eta)} = \boldsymbol{\eta}^*\cdot\mathbf{e}_r.$$

A solution of Eq. (15), without singularities on the axis of the spherical coordinate system, is sought in the form of an expansion in the spherical functions (14). In that case, as seen from the boundary condition (17), to determine the required dependence between $\mathbf{u}(t)$, $\boldsymbol{\eta}(t)$, and $\gamma(t, \theta, \varphi)$ it is sufficient to consider only one component of the total solution, corresponding to the mode with $n = 1$. A solution of Eq. (15) for this mode, satisfying the boundary conditions (16), is

$$u_{1r}^* = \left(1 + a_1\frac{1}{r}G\left(\sqrt{s}r\right) + \frac{a_2}{r^3}\right)\Phi_1^{(u)} + \left(a_1'\frac{1}{r}G\left(\sqrt{s}r\right) + \frac{a_2'}{r^3}\right)\Phi_1^{(\gamma)},$$

$$u_{2r}^* = \left(b_1 + b_2\frac{1}{r}F\left(\sqrt{\nu^{-1}s}\,r\right)\right)\Phi_1^{(u)} + \left(b_1' + b_2'\frac{1}{r}F\left(\sqrt{\nu^{-1}s}\,r\right)\right)\Phi_1^{(\gamma)}, \tag{18}$$

$$p_1^* = \left(-sr + \frac{sa_2}{2}\frac{1}{r^2}\right)\Phi_1^{(u)} + \frac{sa_2'}{2}\frac{1}{r^2}\Phi_1^{(\gamma)},$$

$$p_2^* = -\rho s b_1 r\Phi_1^{(u)} - \rho s b_1' r\Phi_1^{(\gamma)},$$

where the constants $a_1,\ a_2,\ a_1',\ a_2',\ b_1,\ b_2,\ b_1',\ b_2'$ are determined by the relations

$$a_2 = -1 - a_1G\left(\sqrt{s}\right),\ a_2' = -a_1'G\left(\sqrt{s}\right),\ b_1 = -b_2F\left(\sqrt{\nu^{-1}s}\right),$$

$$b_1' = -b_2'F\left(\sqrt{\nu^{-1}s}\right),$$

$$a_1 = -3\exp\left(\sqrt{s}\right)\frac{2 + \beta H\left(\sqrt{\nu^{-1}s}\right)}{3 + \sqrt{s} + \beta H\left(\sqrt{\nu^{-1}s}\right)},$$

$$a_1' = \exp\left(\sqrt{s}\right)\frac{2}{3 + \sqrt{s} + \beta H\left(\sqrt{\nu^{-1}s}\right)},$$

$$b_2 = \frac{3(1+\sqrt{s})}{(3+\sqrt{s}+\beta H(\sqrt{\nu^{-1}s}))(\sqrt{\nu^{-1}s}F'(\sqrt{\nu^{-1}s}) - F(\sqrt{\nu^{-1}s}))},$$
$$b_2' = \frac{2}{(3+\sqrt{s}+\beta H(\sqrt{\nu^{-1}s}))(\sqrt{\nu^{-1}s}F'(\sqrt{\nu^{-1}s}) - F(\sqrt{\nu^{-1}s}))}. \quad (19)$$

The function $\Phi_1(\gamma)$ is generated by expanding γ^* in spherical functions as the corresponding mode with n = 1 of this expansion. The functions F, G, H appearing in (18), (19) are determined as in [1]:

$$F(z) = \left(\frac{\operatorname{sh} z}{z}\right)'_z = \frac{\operatorname{ch} z}{z} - \frac{\operatorname{sh} z}{z^2},\ G(z) = \left(\frac{e^{-z}}{z}\right)'_z = -e^{-z}\left(1+\frac{1}{z}\right)\frac{1}{z},$$
$$H(z) = \frac{z^2 F''(z)}{zF'(z) - F(z)}.$$

Substituting in (17) the solution (18) with account of (19) leads to the relation

$$0 = \left(\frac{1}{2}+\rho\right)s\Phi_1^{(u)} + B^*(s)\Phi_1^{(u)} - C^*(s)\Phi_1^{(\gamma)} + (\rho-1)\Phi_1^{(\eta)}. \quad (20)$$

We multiply the expression in (20) by r and take its gradient. In that case, using an identity which is easily proved, and starting from properties of spherical functions

$$\nabla\left(r\Phi_1^{(u)}\right) = -\mathbf{u}^*,\ \nabla\left(r\Phi_1^{(\eta)}\right) = \eta^*,\ \nabla\left(r\Phi_1^{(\gamma)}\right) = \frac{3}{8\pi}\int_{\Gamma_1}\nabla_\Gamma\gamma^*\,d\Gamma$$

(∇_Γ is the surface gradient operator), we finally reach the relation

$$0 = -\left(\frac{1}{2}+\rho\right)s\mathbf{u}^* - B^*(s)\mathbf{u}^* - C^*(s)\frac{3}{8\pi}\int_{\Gamma_1}\nabla_\Gamma\gamma^*\,d\Gamma + (\rho-1)\eta^*. \quad (21)$$

Here the coefficients B*(s), C*(s) are determined as in [1]:

$$C^*(s) = \frac{3(1+\sqrt{s})}{3+\sqrt{s}+\beta H(\sqrt{\nu^{-1}s})},\ B^*(s) = \frac{3}{2}\left(2+\beta H\left(\sqrt{\nu^{-1}s}\right)\right)C^*(s).$$

If the problem becomes such that one must find the velocity of motion of the drop under the action of known mass and capillary forces, then expression (21) is conveniently rewritten in the form

$$\mathbf{u}^*(s) = \left[-C^*(s)\frac{3}{8\pi}\int_{\Gamma_1}\nabla_\Gamma\gamma^*\,d\Gamma + (\rho-1)\eta^*\right]\left[\left(\frac{1}{2}+\rho\right)s + B^*(s)\right]^{-1} \quad (22)$$

and determining the motion reduces to reconstructing the original from the transform (22).

We turn in (21) to the originals, and assign the relation obtained in that case by the form of Newton's second law

$$\rho\mathbf{u}'(t) = -\frac{1}{2}\mathbf{u}'(t) - \int_0^t b(t-t_1)\mathbf{u}'(t_1)\,dt_1 - \frac{3}{2}\frac{2+3\beta}{1+\beta}\mathbf{u}(t) - \quad (23)$$
$$-\frac{9}{8\pi(1+\rho\sqrt{\nu})}\int_{\Gamma_1}\nabla_\Gamma\gamma(t)\,d\Gamma - \frac{3}{8\pi}\int_0^t c(t-t_1)\left\{\int_{\Gamma_1}\nabla_\Gamma\gamma(t_1)\,d\Gamma\right\}dt_1 + (\rho-1)\eta(t).$$

where, as in [1], the coefficients B*(s) and C*(s) are represented in the following form, taking into account their asymptotic behavior at $s \to \infty$

$$B^*(s) = B^*(0) + sb^*(s),\ C^*(s) = C^*(\infty) + c^*(s),$$
$$B^*(0) = \frac{3}{2}\frac{2+3\beta}{1+\beta},\ C^*(\infty) = \frac{3}{1+\rho\sqrt{\nu}}.$$

Following [1], we note that the asymptotic of originals of the functions $b^*(s)$, $c^*(s)$ thus derived is at $t \to 0$

$$b(t) = \frac{9}{2}\frac{\rho\sqrt{\nu}}{1+\rho\sqrt{\nu}}\frac{1}{\sqrt{\pi t}} + O(1),\ c(t) = \frac{3(\rho\sqrt{\nu}-2)}{(1+\rho\sqrt{\nu})^2}\frac{1}{\sqrt{\pi t}} + O(1),$$

while for $t \to \infty$

$$b(t) = \frac{1}{2}\left(\frac{2+3\beta}{1+\beta}\right)^2\frac{1}{\sqrt{\pi t}} + O\left(t^{-3/2}\right),\ c(t) = -\frac{2+3\beta}{6(1+\beta)^2}\frac{1}{\sqrt{\pi t^3}} + O\left(t^{-5/2}\right).$$

Relation (23) is an integrodifferential equation for motion of the drop.

The expressions are somewhat simplified relative to the case of a drop in the special cases of a solid sphere and of a bubble. Thus, one can write down explicitly the originals $b(t)$, $c(t)$, and the integrodifferential equation reduces to a differential equation. A discussion of these problems can be found in [1]. Everything is similar in the situation considered.

As already noted, Eq. (23) has been written in the form of Newton's second law (in dimensionless form). On the left-hand side of (23) we have the product of the drop mass and its acceleration, while on the right-hand side we have forces acting on the drop. To represent these forces more clearly we transform in (23) to dimensional variables, multiply by $4\pi/3$, and obtain

$$\frac{4\pi}{3}\rho_2 a^3\frac{d\mathbf{u}}{dt} = -\frac{2\pi}{3}\rho_1 a^3\frac{d\mathbf{u}}{dt} - \frac{4\pi}{3}\mu_1 a\int_0^t b\left(\frac{\nu_1(t-t_1)}{d^2}\right)\frac{d\mathbf{u}}{dt}(t_1)\,dt_1 -$$
$$- 2\pi\mu_1 a\frac{2+3\beta}{1+\beta}\mathbf{u} - \frac{3}{2(1+\rho\sqrt{\nu})}\int_{\Gamma_a}\nabla_\Gamma\sigma\,d\Gamma - \frac{\nu_1}{2a^2}\int_0^t c\left(\frac{\nu_1(t-t_1)}{a^2}\right)\times \tag{24}$$
$$\times\left\{\int_{\Gamma_a}\nabla_\Gamma\sigma(t_1)\,d\Gamma\right\}dt_1 + \frac{4\pi}{3}(\rho_2-\rho_1)a^3\mathbf{g},$$

where the preceding notations are used for the dimensional velocity, time, and area element ($d\Gamma = a^2\sin\theta d\theta d\varphi$).

The first term on the right-hand side of (24) is the force resulting from the effect of the associated masses, the second is the Basset force, the third is the Stokes force, the sum of the fourth and the fifth is the capillary force, and the sixth is the sum of the gravity and exclusion forces. We note that, unlike [1], here the given state has a substantial vector character.

We further consider the case in which the gravitational field $\mathbf{g}(t)$ and the distribution of the surface tension coefficient $\sigma(t, \theta, \varphi)$ at $t \to \infty$ lead to some stationary values $\mathbf{g}$ and $\sigma(\theta, \varphi)$, i.e.,

$$\lim_{s\to 0} s\mathbf{g}^*(s) = \mathbf{g},\ \lim_{s\to 0} s\sigma^*(s, \theta, \varphi) = \sigma(\theta, \varphi). \tag{25}$$

In that case it is interesting to find, for example, the limiting expressions for the capillary force and the velocity of drop motion. Multiplying the transforms of these quantities [in dimensionless form this means (22) and the capillary terms in (21)] by s and taking the limit $s \to 0$, with account of (25) we obtain (in dimensional form)

$$\mathbf{F}_k = -\frac{1}{2(1+\beta)}\int_{\Gamma_a}\nabla_\Gamma\sigma\,d\Gamma; \tag{26}$$

$$\mathbf{u} = \frac{2(1+\beta)(\rho_2-\rho_1)a^2}{3(2+3\beta)\mu_1}\mathbf{g} - \frac{1}{4\pi a\mu_1(2+3\beta)}\int_{\Gamma_a}\nabla_\Gamma\sigma\,d\Gamma. \tag{27}$$

The expression for the stationary capillary force (26) coincides totally with that obtained in [2].

For the skilled reader we note that, in principle, the basic results obtained here by means of a quite simple generalized discussion, based on the linearity of the problem and on the importance of only a single mode of the solution for describing the motion of the drop center of mass, could be written down directly, starting from Eqs. (5.1), (5.2) of [1], and treating independently the motion along each of the three mutually perpendicular coordinate axes. However, the authors have preferred not to pursue this path.

If the distribution of the surface tension coefficient $\sigma(\theta, \varphi)$ and the gravitational field $\mathbf{g}$ are from the very beginning constant in time, the original expression (22) can be analyzed in more detail in several limiting cases (see [4, Sec. 5, Chap. 4]).

Further, following Landau and Lifshitz ([5, Problem 7, Sec. 24]), consider the problem without initial conditions. As applied to the situation investigated in the present paper, it can be formulated as follows: find the resistance force (or, better here, the hydrodynamic force) acting on the drop, if the drop velocity and the distribution of the surface tension coefficient are known functions of time $\mathbf{u}(t_1)$ and $\sigma(t_1, \theta, \varphi)$ at $-\infty < t_1 < t$. The answer can be written down directly on the basis of the right-hand side of (24):

$$\mathbf{F} = -\frac{2\pi}{3}\rho_1 a^3 \frac{d\mathbf{u}}{dt} - \frac{4\pi}{3}\mu_1 a \int_{-\infty}^{t} b\left(\frac{\nu_1 (t - t_1)}{a^2}\right) \frac{d\mathbf{u}}{dt}(t_1)\, dt_1 - 2\pi\mu_1 a \frac{2+3\beta}{1+\beta}\mathbf{u} - \frac{3}{2(1+\rho\sqrt{\nu})}\int_{\Gamma_a} \nabla_\Gamma \sigma\, d\Gamma - \frac{\nu_1}{2a^2}\int_{-\infty}^{t} c\left(\frac{\nu_1 (t - t_1)}{a^2}\right)\left\{\int_{\Gamma_a} \nabla_\Gamma \sigma(t_1)\, d\Gamma\right\} dt_1. \tag{28}$$

Relationship (28) is in some sense the generalization of the corresponding equation in [5].

If the nonconstancy of the surface tension is related to inhomogeneous fields of temperature, concentration, etc., one can carry out further specification of the results obtained, finding these fields and knowing the shape of the thermodynamic dependence of the surface tension coefficient. Here it is substantial that the Peclet numbers are small, i.e., the distributions of temperature, concentration, etc., implying also that the surface tension is independent of the fluid motion or else our treatment is not uniformly correct. Such a capability of the results of the present study can be found, for example, in the results of [1], as well as the corresponding results for nonstationary thermocapillary drop motion under the action of radiation (which was investigated in [3] within the stationary statement) and for nonstationary thermocapillary drop motion in an external constant temperature gradient for a nonlinear temperature dependence of the surface tension coefficient (which was treated in [6] within the quasistationary approximation).

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